

## 9. TRANSMISSION OF SOUND THROUGH STRUCTURES

### 9.1 Basic Definitions

A typical noise control application involves a combination of absorption of sound and transmission of sound energy by a variety of airborne and structure-borne paths.

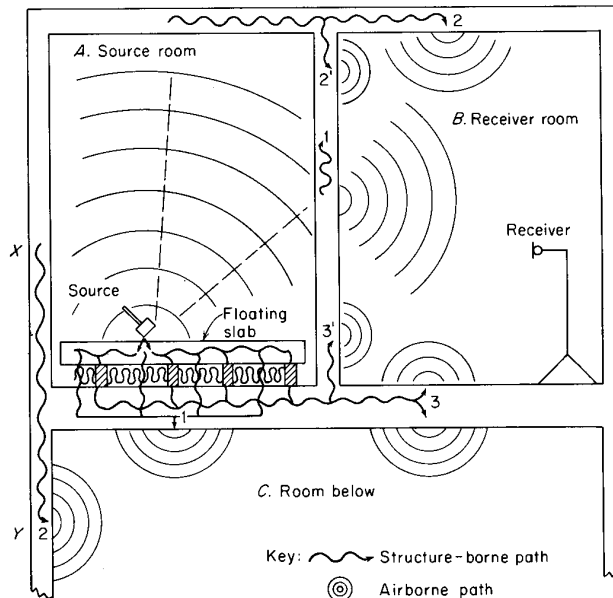


Figure 1. Sound transmission paths between a room containing a noise source and adjacent rooms

Some important definitions and concepts:

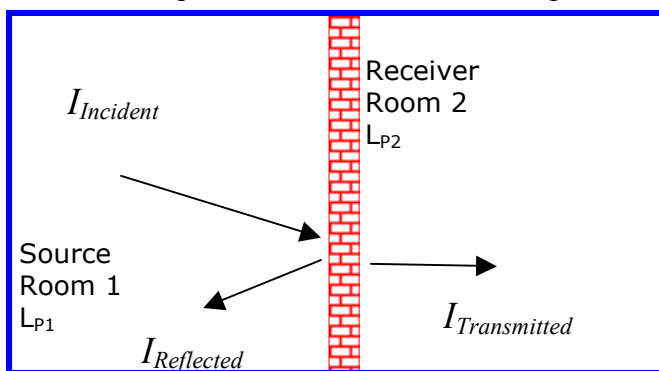


Figure 2. When sound strikes a partially absorbing partition between two rooms, some is reflected back into room, some transmits into adjacent room

**Transmission Coefficient  $\tau$** , for walls

$$\tau = \frac{I_{\text{Transmitted}}}{I_{\text{Incident}}}$$

(  $\tau$  is a frequency-dependent physical property of the material)

**Sound Transmission Loss**

STL = the log ratio of the incident energy to the transmitted energy

$$\text{STL} = 10 \log 1/\tau \quad \text{Equation 1}$$

A tabulation of transmission loss for common materials is included in the Appendix to this section (from Table 8.1, ref Bies and Hansen). A perfectly reflecting material has a transmission coefficient of 0 ( $\text{STL} = \infty$ ), while the transmission coefficient of an opening is 1.0

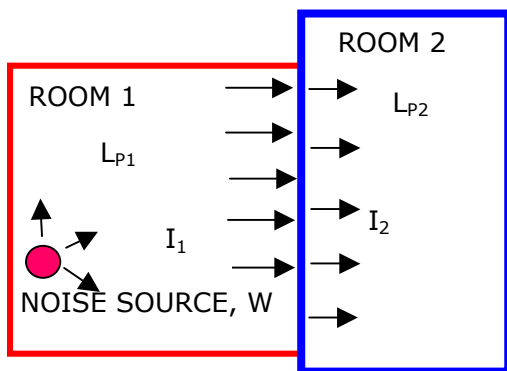
(STL=0). It should be noted that typical materials tend to be better at blocking higher frequencies. Transmission loss can be measured directly (but not easily) by mounting a test panel between two reverberation rooms and measuring the sound pressure levels on each side.

Other commonly used metrics to describe sound transmission include:

**NR** = Noise Reduction =  $L_{P1} - L_{P2}$  (easy to measure)      **Note: NR  $\neq$  STL !**

**IL** = Insertion Loss = change in sound levels with and without the barrier or treatment in place (easy to measure)

## 9.2 Relation Between $L_{P1}$ and $L_{P2}$



The sound power incident on the left side of the wall is, (assuming a diffuse sound field):

$$W_I = I_I S_W \quad S_W = \text{Area of the common wall}$$

$$I_I = \text{Intensity incident on wall}$$

$$I_2 = \text{Intensity transmitted to room 2}$$

and since  $I_I = 1/4$  the intensity in a plane wave, the sound power striking the left side of the wall is:

$$W_I = I_I S_W = \frac{\langle p_1^2 \rangle}{4\rho c} S_W$$

Figure 3. Sound transmission between two rooms, the area of the common wall =  $S_W$

In the receiving room (Room 2), we know that whatever power comes through the common wall, must eventually be absorbed in that room. We will call the power coming through the wall  $W_2$ :

$$W_2 = I_2 S_2 \alpha_2 \quad \text{where } S_2 = \text{total surface area of receiving room}$$

$$\alpha_2 = \text{room average absorption for receiving room}$$

and assuming a diffuse field in receiving room,  $I_2 = \langle p_2^2 \rangle S_2 \alpha_2 / (4\rho c)$

Using the definition of  $\tau$ :

$$\tau = \frac{I_2}{I_1} = \frac{W_2}{W_1} = \frac{\langle p_2 \rangle^2 S_2 \alpha_2}{\langle p_1 \rangle^2 S_W}$$

Take the log and rearrange:  $STL = 10 \log_{10} \left( \frac{1}{\tau} \right) = 10 \log_{10} \frac{\langle p_1 \rangle^2 / p_{REF}^2}{\langle p_2 \rangle^2 / p_{REF}^2} + 10 \log_{10} \frac{S_W}{S_2 \alpha_2}$

or more simply:  $STL = L_{P1} - L_{P2} + 10 \log_{10} \frac{S_W}{S_2 \alpha_2}$       Equation 2

Now we have a very useful expression that will tell us the sound level in the receiver room 2.

### 9.3 Sound Levels in Source Room

Meanwhile, back in the noise source room, Room 1:

$$L_{P1} = L_W + 10 \log_{10} \left[ \frac{Q_\theta}{4\pi r^2} + \frac{4}{R} \right] \quad R = \frac{\overline{S\alpha_{ST}}}{1 - \alpha_{ST}}$$

If we neglect the direct field portion, (ok approximation if room is not too absorptive and you are far from the noise sources), then:

$$L_{P1} = L_W + 10 \log_{10} \frac{4}{R}$$

Substituting into equation 2:

$$L_W + 10 \log_{10} \frac{4}{R} + 10 \log_{10} \frac{S_W}{S_2 \alpha_2} - STL = L_{P2} \quad \text{Equation 3}$$

Look at trends to see if this equation makes physical sense:

$L_{P2}$  decreases

- if  $R$  increases (more absorption in room 1)
- if  $STL$  increases (more transmission loss, i.e. a better wall)
- if  $\alpha_2$  increases (more absorption in room 2)
- if  $S_w$  decreases (less common area, transmitted power is proportional to Intensity striking the wall times wall area)

**HW Problem 1.** In addition to a barrier material, an absorbing layer ( $\alpha=.90$ ) is to be used on the wall between a source room and a receiver room. The barrier material has an absorption coefficient of  $\alpha=.10$ . The TL of the composite wall is 43. Each room is a 5 meter cube. What is the difference between placing the absorbing layer on the source side versus the receiver side of the wall? (answer: each case results in the same levels in room 2, however putting it on the inside also decreases the level inside room 1)

## 9.4 Noise Enclosures

An enclosure around a noise source is just a special case of the two room problem, where the enclosure is one room, and the surrounding space is the second room (Figure 4). It can be shown that the insertion loss for this case is:

$$IL = L_{P_2}' - L_{P_2} \cong 10 \log \frac{\alpha}{\tau} \quad \text{Equation 4}$$

Where:  $L_{p2}$  = SPL without enclosure  
 $L_{p2}$  = SPL with enclosure  
 $\alpha$  = effective absorption coefficient within enclosure  
 $\tau$  = effective transmission coefficient  
of enclosure walls

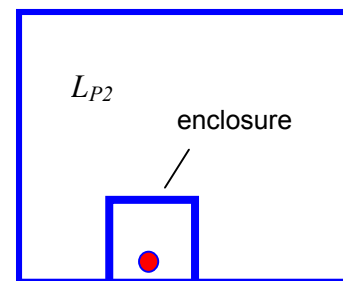


Figure 4. Enclosure around a noise source

**H.W. Problem 2:** Verify equation 4. List all assumptions that are made in the derivation

**H.W. Problem 3:** A 1x1x1 meter enclosure is placed around a noise source in a 5x5x5 meter room. In addition to a barrier material, an absorbing layer ( $\alpha=.90$ ) is used on the walls of the enclosure. The TL of the composite wall is 43. The barrier material has an absorption coefficient of  $\alpha=.10$ . Should the absorbing layer be on the outside or inside surface of the enclosure for maximum effect? (answer: putting it on the inside results in 8 dB lower levels in room)

## 9.5 What is the difference between an absorbing material and a barrier material?

The two important noise-related quantities of a material are:

- Ability to absorb acoustic energy -  $\alpha$
- Ability to reflect or block sound energy - STL or  $\tau$

Good absorbing materials allow sound pressure fluctuations to enter their surface and dissipate energy by air friction. They are generally porous and lightweight, such as fiberglass, open cell foam, or acoustical ceiling tiles. Good barrier materials reflect sound, and are dense and non-porous (concrete, lead, steel, brick, glass, gypsum board). In general, a single homogeneous material will not be both a good absorber and a barrier. As shown in Table 1, fiberglass insulation makes a terrible barrier, and a sealed concrete wall has virtually no absorption. To get the best of both worlds, it is common to see an absorbing layer laminated to a barrier material, for instance a layer of gypsum board and a layer of fiberglass, or loaded vinyl laminated to open cell foam.

Table 1. Comparison of various material noise properties at 1000 Hz

Material	Absorption $\alpha$	Transmission $\tau$
Concrete Cinder Block (painted)	.07 very low	.0001 (STL=40) high
2" Fiberglass	.90 high	~1.0 very low

## 9.6 Sound Transmission Class

Sound Transmission Class (STC) is a single number rating of transmission loss which is popular with architects. It is obtained by fitting the closest standard contour to the actual TL vs frequency data for the material (see Figure 5, ref Bies and Hansen). The STC is determined by comparing the set of transmission losses at all 16 1/3 octave center frequencies to a set of standard contours as described in ASTM Standard E413-70T. The TL curve must fit the standard contour in such a way that in no event is the TL curve more than 8 dB below the STC contour at any frequency, and the sum of the deviations of the TL values which are below the contour shall not exceed 32 dB. The highest contour to which the specimen TL curve can satisfy these requirements is used as the STC curve. Table 2 relates STC with the hearing quality. The STC of various composite frame wall constructions is shown in Figure 15.6 in the Appendix. The standard STC contours are shown in Table 3.

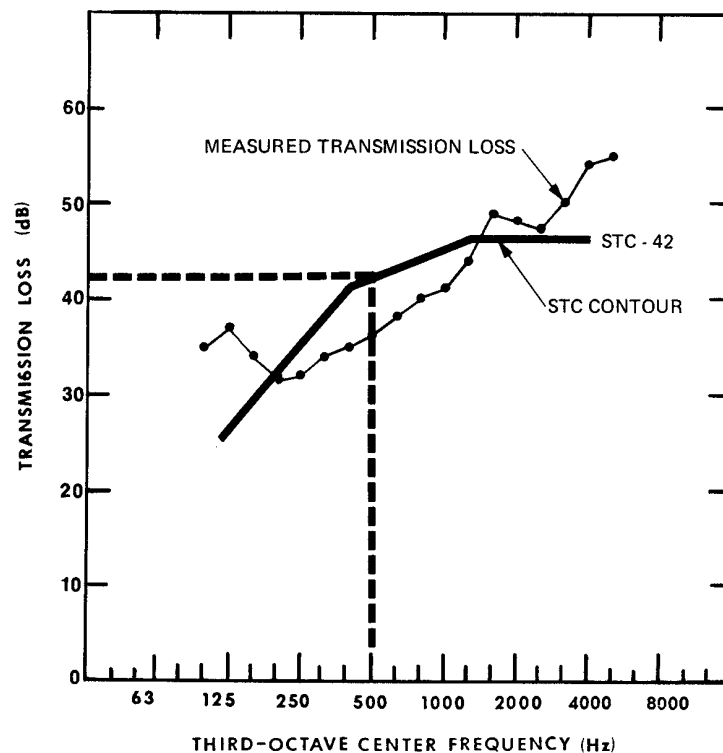


Figure 5. STC rating is determined by adjusting the standard contours to the measured TL values and reading the contour intercept at 500 Hz.

Table 2. Typical hearing quality for a wall of rated sound transmission class (STC)

Sound Transmission Class (STC)	Hearing Quality Through Wall
25	Normal speech understood quite easily and distinctly through wall
30	Loud speech understood fairly well, normal speech heard but not understood
35	Loud speech heard but not intelligible
40	Onset of "privacy"
42	Loud speech audible as a murmur
45	Loud speech not audible, 90% of statistical population not annoyed
50	Very loud sounds such as musical instruments or a stereo can be faintly heard, 99% of population not annoyed

Table 3. Standard STC contours (note: a particular contour is identified by its TL value at 500 Hz) from ASTM E413

Note: A particular contour is identified by its *TL* value at 500 Hz.  
(From ASTM E413)

Hz	125	160	200	250	315	400	500	630	800	1,000	1,250	1,600	2,000	2,500	3,150	4,000
Sound Transmission Loss	44	47	50	53	56	59	60	61	62	63	64	64	64	64	64	64
	43	46	49	52	55	58	59	60	61	62	63	63	63	63	63	63
	42	45	48	51	54	57	58	59	60	61	62	62	62	62	62	62
	41	44	47	50	53	56	57	58	59	60	61	61	61	61	61	61
	40	43	46	49	52	55	56	57	58	59	60	60	60	60	60	60
	39	42	45	48	51	54	55	56	57	58	59	59	59	59	59	59
	38	41	44	47	50	53	54	55	56	57	58	58	58	58	58	58
	37	40	43	46	49	52	53	54	55	56	57	57	57	57	57	57
	36	39	42	45	48	51	52	53	54	55	56	56	56	56	56	56
	35	38	41	44	47	50	51	52	53	54	55	55	55	55	55	55
	34	37	40	43	46	49	50	51	52	53	54	54	54	54	54	54
	33	36	39	42	45	48	49	50	51	52	53	53	53	53	53	53
	32	35	38	41	44	47	48	49	50	51	52	52	52	52	52	52
	31	34	37	40	43	46	47	48	49	50	51	51	51	51	51	51
	30	33	36	39	42	45	46	47	48	49	50	50	50	50	50	50
	29	32	35	38	41	44	45	46	47	48	49	49	49	49	49	49
	28	31	34	37	40	43	44	45	46	47	48	48	48	48	48	48
	27	30	33	36	39	42	43	44	45	46	47	47	47	47	47	47
	26	29	32	35	38	41	42	43	44	45	46	46	46	46	46	46
	25	28	31	34	37	40	41	42	43	44	45	45	45	45	45	45
	24	27	30	33	36	39	40	41	42	43	44	44	44	44	44	44
	23	26	29	32	35	38	39	40	41	42	43	43	43	43	43	43
	22	25	28	31	34	37	38	39	40	41	42	42	42	42	42	42
	21	24	27	30	33	36	37	38	39	40	41	41	41	41	41	41
	20	23	26	29	32	35	36	37	38	39	40	40	40	40	40	40
	19	22	25	28	31	34	35	36	37	38	39	39	39	39	39	39
	18	21	24	27	30	33	34	35	36	37	38	38	38	38	38	38
	17	20	23	26	29	32	33	34	35	36	37	37	37	37	37	37
	16	19	22	25	28	31	32	33	34	35	36	36	36	36	36	36
	15	18	21	24	27	30	31	32	33	34	35	35	35	35	35	35
	14	17	20	23	26	29	30	31	32	33	34	34	34	34	34	34

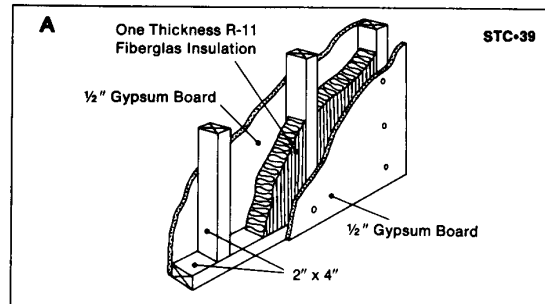
## 9.7 Multi-Layer Panels - Constructions for High STC

Examples of multiple layer constructions to achieve high STC are shown in the following figures. The fundamental principles are to minimize any direct mechanical connection between the two surfaces of the wall, and to have no openings or leaks. Problem areas for leaks are air vents, around doors, electrical outlets or pipe penetrations.

**A STC-39** Single wood studs 16" o.c.; single layer ½" gypsum board each side; one thickness R-11" Fiberglass insulation

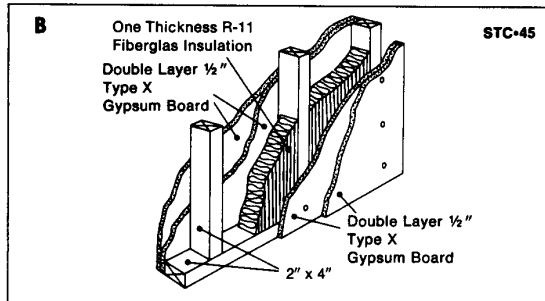
**STC-35** Single wood studs 16" o.c.; single layer ½" gypsum board each side; no insulation

\*Nominal 3½" Thick



**B STC-45** Single wood studs 16" o.c.; double layer ½" type x gypsum board each side; one thickness R-11 Fiberglass insulation

**STC-39** Single wood studs 16" o.c.; double layer ½" gypsum board each side; no insulation

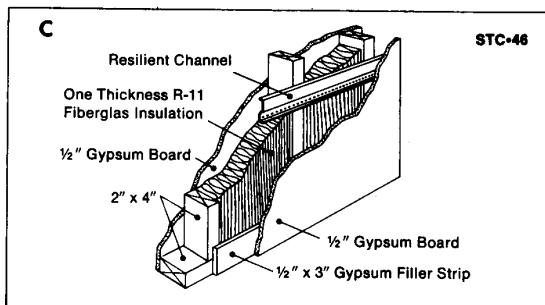


**C STC-46** Single wood studs with resilient channel; single layer ½" gypsum board each side; one thickness R-11 Fiberglass insulation

**STC-40** Single wood studs with resilient channel; single layer ½" type x gypsum board each side; no insulation

**STC-39** Single wood studs with resilient channel; single layer ½" gypsum board each side; no insulation

**STC-56** Single wood studs with resilient channel; double layer ½" gypsum board each side; one thickness R-11 Fiberglass insulation



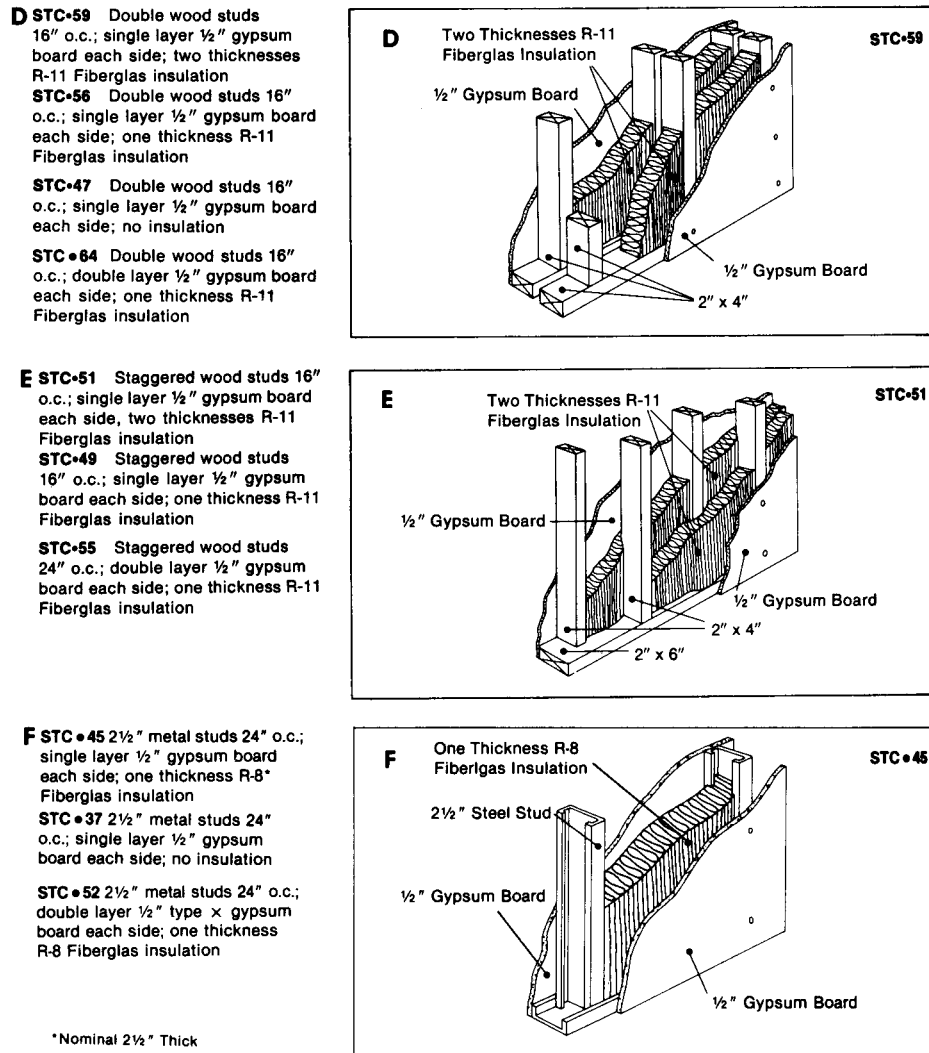


Figure 6 Construction details of frame walls for high STC (courtesy of Owens Corning)

Similar considerations apply to floors and ceilings. Additionally, floors are rated by their Impact Insulation Class, (IIC). Resilient layers, or carpet are used to insulate the transmission of impact noise (such as footsteps). Layered floor constructions are shown in Figure 7.

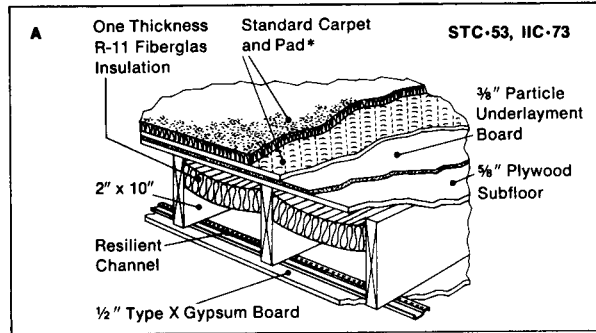


**A. Wood Floors (all on 2" x 10" joists)**

**STC-53, IIC-73** Carpet and pad; particle board surface; plywood subfloor; single layer 1/2" type x gypsum ceiling on resilient channel; one thickness R-11 Fiberglas insulation

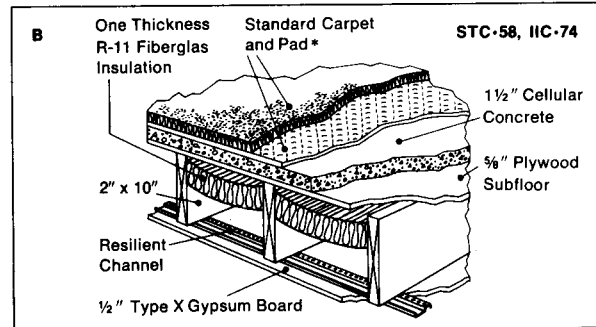
**STC-43, IIC-60** Carpet and pad; particle board surface; plywood subfloor; single layer 1/2" type x gypsum ceiling attached directly to joists; one thickness R-11 Fiberglas insulation

**STC-42, IIC-60** Carpet and pad; particle board surface; plywood subfloor; single layer 1/2" type x gypsum ceiling attached directly to joists; no insulation

**B. Lightweight (Cellular) Concrete Floors (1 1/2" thick over 5/8" plywood subfloor on 2" x 10" joists)**

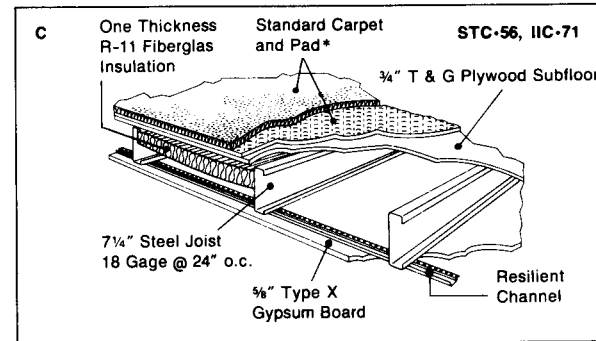
**STC-58, IIC-74** Carpet and pad; single layer 1/2" type x gypsum ceiling attached to joists by resilient channel; one thickness R-11 Fiberglas insulation

**STC-47, IIC-59** Carpet and pad; single layer 1/2" type x gypsum ceiling attached directly to joists; no insulation



**C. Steel Joist (7 1/4" x 18 gage, @ 24" o.c.)** **STC-56, IIC-71** Carpet and pad, 3/4" T & G plywood sub floor, 5/8" type x gypsum board attached to ceiling joists by resilient channel; one thickness R-11 Fiberglas insulation

**STC-43, IIC-57** Carpet and pad; 3/4" T & G plywood sub floor, 5/8" type x gypsum board attached directly to the joists; no insulation



\*The carpet and pad were selected to represent the type and quality commonly specified for new construction and was 20-oz. textured loop nylon carpet backed by a 40-oz. all-hair pad.

Figure 7 Construction details for floor and ceiling systems that control impact sound (Courtesy of Owens Corning)

## 9.8 Composite Walls

If a wall or partition is not of uniform construction, its effective transmission loss can be determined by summing over the  $n$  different surfaces in the same manner that we determined a room-averaged absorption coefficient:

$$\tau_{\text{composite}} = \frac{\sum_{i=1}^n \tau_i S_i}{\sum_{i=1}^n S_i} \quad \text{Equation 5}$$

where  $\tau_i$  = transmission coefficient of  $i^{\text{th}}$  surface, having area  $S_i$

Example: a wall with door and window

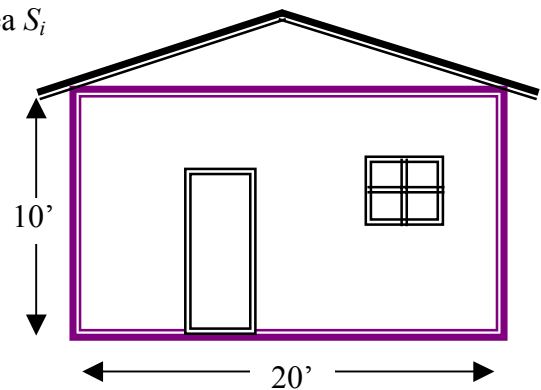
Surface	Area	STL	$\tau$
Door	3' x 7' = 21 ft <sup>2</sup>	20	.01
Window	2' x 4' = 8 ft <sup>2</sup>	30	.001
Wall (8" concrete)	200-29=171 ft <sup>2</sup>	50	.00001

STL for the composite wall =  
(29.6)

What if you use a wall material with STL = 40?  
(29.3)

What if there is a vent in the wall of 1 ft<sup>2</sup>?  
(23)

What conclusions can you draw from this example?



## 9.9 Transmission Through Panels

The transmission loss of an infinite homogeneous panel is shown in Figure 8.

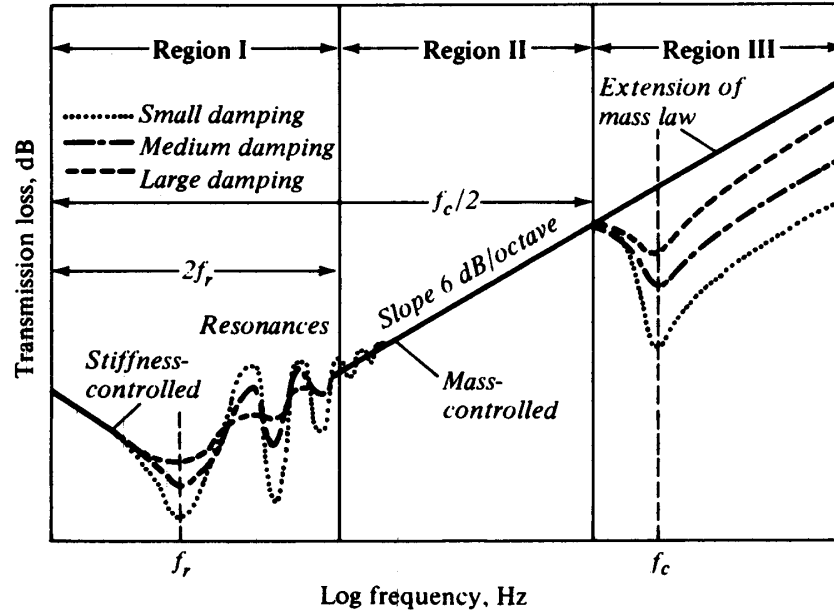


Figure 8. Theoretical Transmission Loss for an infinite homogeneous panel

STL or  $\tau$  are highly dependent on frequency. The STL behavior can be divided into three basic regions. In Region I, at the lowest frequencies, the response is determined by the panel's static stiffness. Depending on the internal damping in the panel, resonances can also occur which dramatically decrease the STL. Calculation of natural frequencies and modes shapes for panels is discussed in Section 9.9.

In Region II (mass-controlled region), the response is dictated by the mass of the panel and the curve follows a 6dB/octave slope. Doubling the mass, or doubling the frequency, results in a 6 dB increase in transmission loss.

In this region, the normal incidence transmission loss can be approximated by:

$$TL_0 = 10 \log \left[ 1 + \left( \frac{\omega \rho_s}{2 \rho c} \right)^2 \right] \quad \text{dB} \quad \text{Equation 6}$$

where:  $\omega$  = sound frequency (rad/sec)

$\rho c$  = characteristic impedance of medium (415 rayls for air at standard temperature and pressure)

$\rho_s$  = mass of panel per unit surface area

The random incidence transmission loss is:

$$TL \approx TL_0 - 10 \log(.23 TL_0) \quad \text{dB} \quad \text{Equation 7}$$

In Region III, coincidence between the sound wavelength and the structural wavelength again decrease the STL. Coincidence is further described in Section 9.10.

The actual behavior of some common building materials, shown in Figure 9, follows the same basic trends. It is most desirable to use a barrier material in its mass-controlled region.

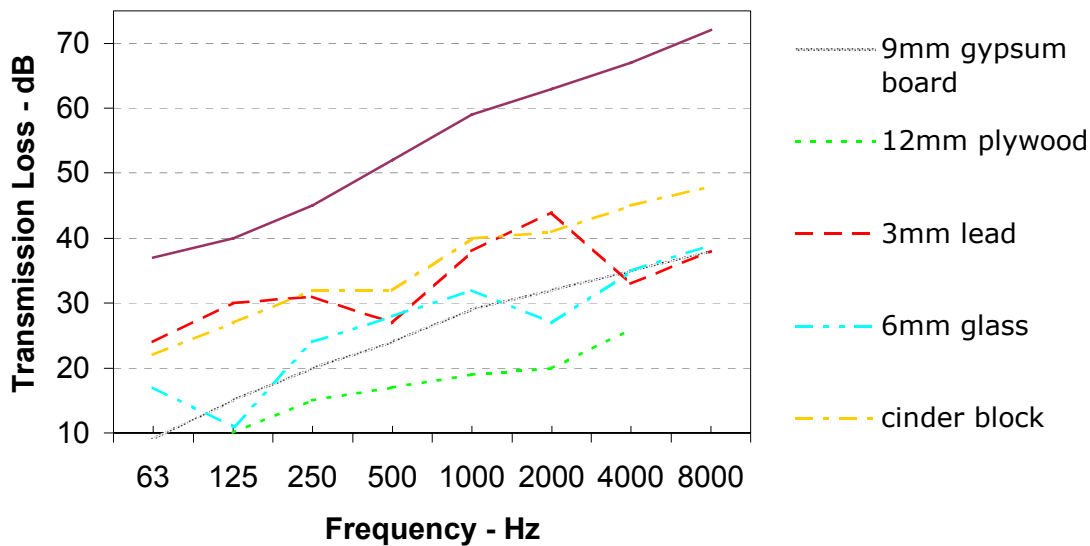


Figure 9. Sound Transmission Loss of Typical Building Materials (data from Table 8.1, Bies and Hansen)

Properties which make for a good barrier material include:

- High density (gives high STL in mass-controlled region)
- Low bending stiffness (ideally want resonant frequencies below range of human hearing)
- High internal damping (prevents resonant modes from “ringing”)

The ideal material for high STL is sheet lead, which has both high density and low stiffness. Unfortunately, due to environmental health concerns, lead can no longer be used. For the same reasons, gypsum board is a good barrier material and is more effective than plywood (which is stiffer and not as dense as gypsum board). Loaded vinyl, or vinyl impregnated with metal filings, is a common material for high STL.

## 9.10 Panel Natural Frequencies and Mode Shapes

Knowledge of a panel's natural frequencies and mode shapes is extremely helpful. It allows us to predict and hopefully avoid having excitation frequencies (harmonic forces generated by a machine) coincide with structural resonances. Knowledge of the mode shape is useful because:

- provides guidance for stiffening a structure in order to change its natural frequencies;
- provides guidance for adding laminated damping material to limit the response at resonance
- the mode shape determines the radiation efficiency of the panel – if the structural wavelength is larger than the acoustic wavelength, the panel will radiate very efficiently (discussed further in section 9.10)

Natural frequencies and mode shapes can be predicted by closed form solution of Euler's thin plate approximation for some regular geometries including:

- a) beams
- b) rectangular plates
- c) triangular plates
- d) circular plates
- e) rings

The natural frequencies of simply-supported, rectangular, thin, isotropic plate are described by a simple equation. Analytical solutions for other boundary conditions are not nearly so simple.

$$f(n_x, n_y) = \frac{\pi}{2} \sqrt{\frac{Eh^2}{12\rho}} \left[ \left( \frac{n_x}{L_x} \right)^2 + \left( \frac{n_y}{L_y} \right)^2 \right] \quad \text{Eq. 8}$$

where:

$E$  = Young's modulus

$h$  = plate thickness

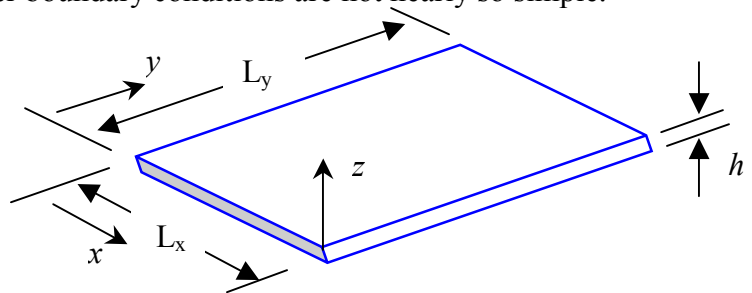
$\rho$  = mass density/unit volume

$n_x$  =  $x$  mode index, # of half sine waves along  $x$  axis

$n_y$  =  $y$  mode index, # of half sine waves along  $y$  axis

$L_x$  = plate width in  $x$  direction

$L_y$  = plate width in  $y$  direction



The modeshape of the simply supported plate consists of sinusoidal segments:

$$z(x, y) = A \sin \frac{n_x \pi x}{L_x} \sin \frac{n_y \pi y}{L_y} \quad \text{Equation 9}$$

where  $z(x, y)$  is the transverse displacement at position  $(x, y)$

**Example:** Calculate the natural frequency of the 1,2 mode of a 30" square steel plate which is 0.125" thick. Material properties for steel:  $E = 30\text{e}6 \text{ lb/in}^2$   $\rho = .283/386 \text{ lb sec}^2/\text{in}^4$

**Answer:**

$$f_{1,2} = \frac{\pi}{2} \sqrt{\frac{30\text{e}6(.125)^2}{12(.283/386)}} \left[ \left( \frac{1}{30} \right)^2 + \left( \frac{2}{30} \right)^2 \right] = 63.7 \text{ Hz}$$

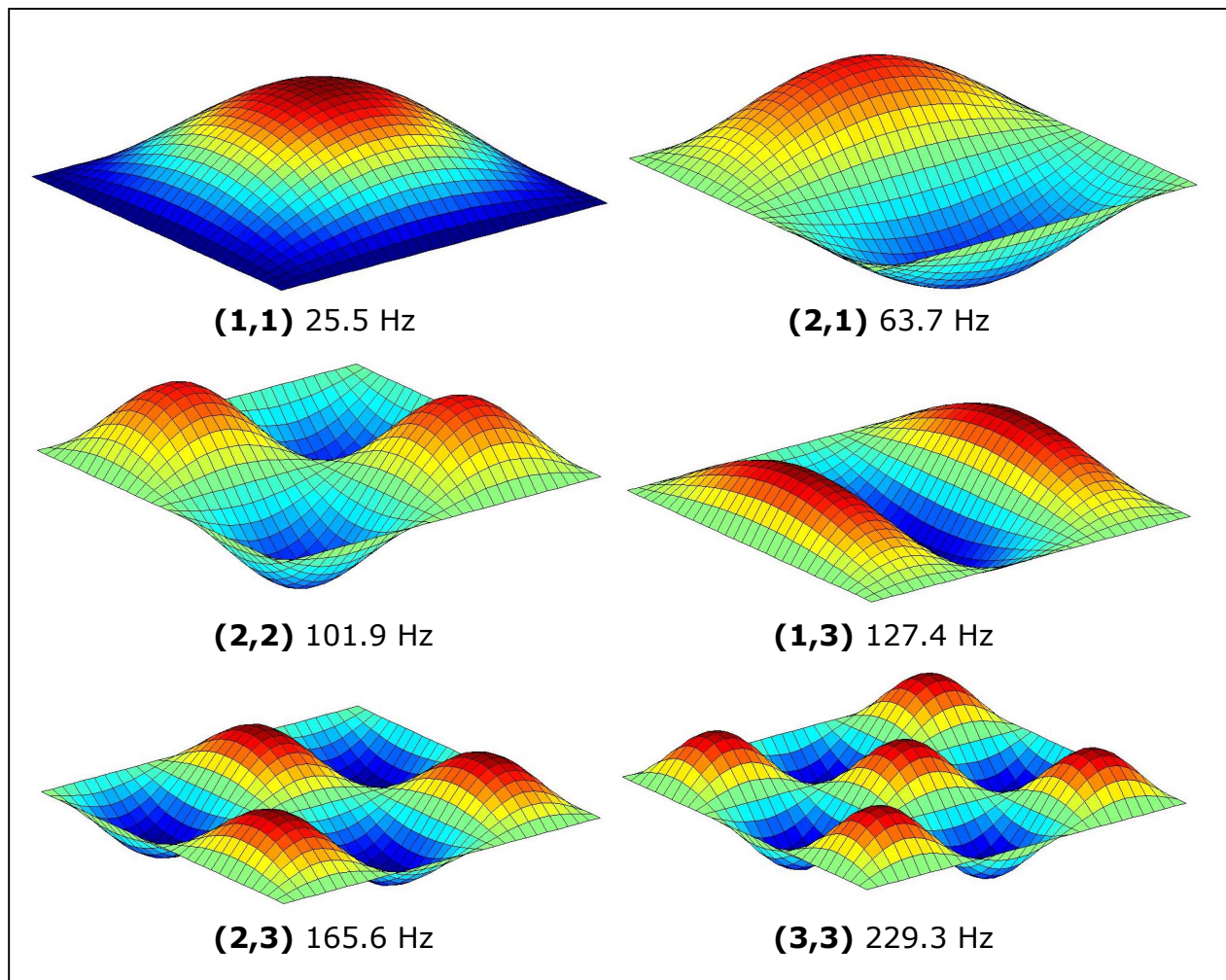


Figure 10. The first six mode shapes of a square, simply supported, isotropic plate (surface plots from MATLAB) Plate dimensions: 30" square, 1/8" thick, steel

A plate is an example of a continuous system, which has an infinite number of mode shapes and natural frequencies. The lowest frequency is commonly called the fundamental frequency. The higher natural frequencies are **not** integer multiples of the fundamental frequency. The first 6 mode shapes of a square simply supported plate are illustrated in Figure 10. The modes are numbered according to how many half sine waves are found in each direction. Hence, the 1,2 mode would have one  $\frac{1}{2}$  sine wave in the x-direction and two  $\frac{1}{2}$  sine waves in the y-direction. The locations (in this case straight lines) where the displacement is always zero are called **nodes**. Note that the square plates have so-called "degenerate" modes, where two mode shapes (for example the 2,1 and 1,2) have exactly the same frequency. In real life, nothing is ever perfectly square, so you end up with two very closely spaced modes.

A very complete compilation of formulas and tables for calculating the natural frequencies of a variety of shapes can be found in Section 3.4 of NASA Sonic and Vibration Environments for Ground Facilities – A Design Manual, Wylie Labs NAS8-11217.

For complicated geometries, discretized numerical solutions, such as Finite Element Analysis (FEA) are commonly used. As an example, the ANSYS program was used to predict the frequencies and mode shapes of a clamped edge 24" x 30" x .125" plate. 80 thin plate elements were used. The predicted frequencies are compared to experimental data, and to the analytical solution (using the NASA tables), in Table 3. Note that FEA solutions typically over-predict the natural frequencies. The agreement improves if more elements are used to discretize the structure.

Table 3. Comparison of experimental, FEA (ANSYS program), and analytical results for a 30" x 24" x .125" steel plate with clamped edges

<b>Mode</b>	<b>1,1</b>	<b>2,1</b>	<b>1,2</b>	<b>3,1</b>
<b>Experimental Natural Freq - Hz</b>	62.5	95	125	160
<b>FEA - Hz</b>	64.5	112	148	190
<b>Analytical - Hz</b>	63	111	144	188

Experimental methods to find natural frequencies and mode shapes include modal analysis, where a known input force is applied by a shaker, or an impact, and the frequency response is measured using FFT techniques. Mode shapes can also be determined visually by an antique, but clever technique - Chladni patterns. In this method, sand spread on a plate, which is vibrating at resonance, collects at the nodal lines (see Figure 11).



Figure 11. Chladni patterns for violin top and back plates (Cover of Scientific American Magazine, October 1981)



## 9.11 Coincidence Effects

Getting back to Region III of Figure 6, we see a pronounced dip in the transmission loss curve. This occurs when the wavelength of sound in air coincides with the structural wavelength. At this frequency (and above), efficient radiation of sound occurs. For a homogeneous, infinite plate, this “critical frequency” is:

$$f_c = \frac{5.2 \times 10}{h} \frac{\rho}{E} \quad \text{Hz} \quad \text{Equation 10}$$

where:  $\rho$  = weight density (lb/in<sup>3</sup>)  
 $h$  = plate thickness (inches)  
 $E$  = elastic modulus (psi)

For glass, steel or aluminum (all have similar  $\rho/E$ ), this simplifies to:  $f_c \approx \frac{500}{h}$

For plywood  $f_c \approx \frac{790}{h}$ . Drywall typically exhibits a 5-10 dB dip in TL at ~500 Hz.

The ideal barrier material has high density and low bending stiffness (i.e. very limp). In the old days, lead sheet or leaded vinyl were widely used. Today, loaded vinyl (impregnated with non-lead metal) is a good choice. Dense, limp materials tend to push the coincidence frequency upward and out of the range of interest. Coincidence dips are a problem for materials with low internal damping and high bending stiffness (such as metals or glass).

The radiation efficiency of a simply supported square plate in the vicinity of coincidence is shown in Figure 10 (ref. Wallace, 1972).

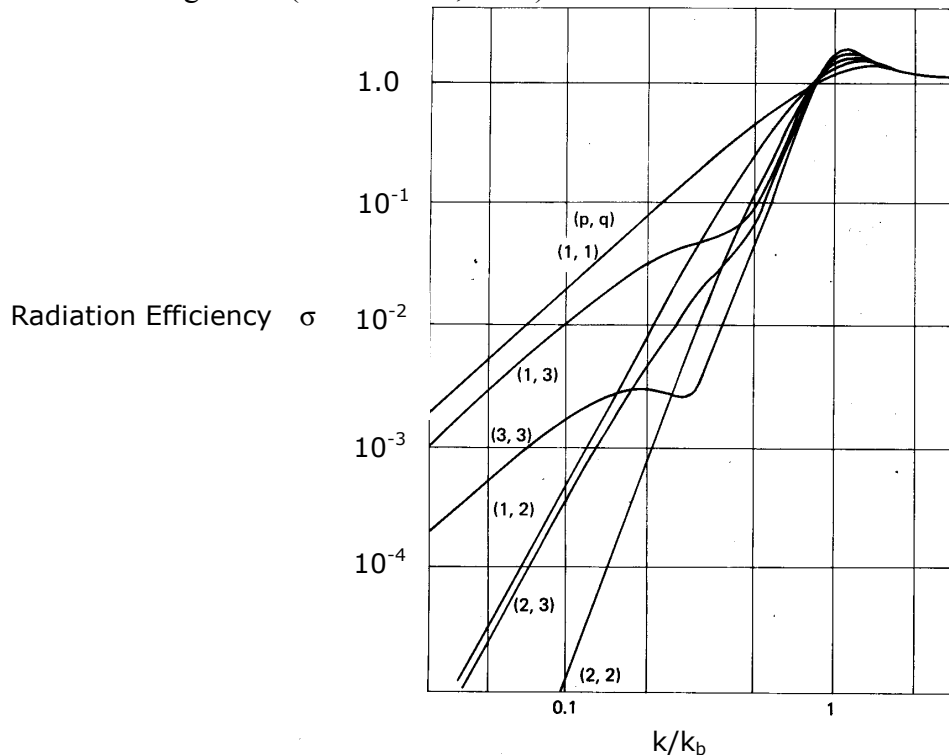


Figure 12. Radiation efficiency for modes of a simply supported square panel



Radiation efficiency,  $\sigma$  is defined as the ratio of the actual energy radiated ( $W$ ) by the structure to the amount of energy that would be radiated by a circular piston of the same area ( $S$ ) and having the same mean square normal velocity  $\langle V_N^2 \rangle$ , and having a diameter much larger than the wavelength in air:

$$\sigma = \frac{W}{\frac{1}{2} \rho c S \langle V_N^2 \rangle} \quad \text{Equation 11}$$

The horizontal axis of Figure 12,  $k/k_b$ , is the ratio between structural and acoustic wave numbers (ratio of structural wavelength to the acoustic wavelength). The plate dimensions are  $a, b$  and  $n, m$  are the number of  $1/2$  sine waves in each plate dimension.

$$k = \omega / c = \frac{2\pi}{\lambda_{air}} \quad k_b = \sqrt{\left(\frac{n\pi}{a}\right)^2 + \left(\frac{m\pi}{b}\right)^2} = \frac{2\pi}{\lambda_{plate}} \quad \text{Equation 12}$$

All modes become very efficient radiators near coincidence. Below coincidence, some radiation still occurs, predominantly from the corners and edges of the plate as shown in Figure 13. Adjacent peaks and valleys on the surface cancel each other, leaving just the edges to radiate. The odd modes (such as 1,1 1,3 ...) radiate better than the even modes (2,2 2,4 ...) because the uncanceled portions are in phase with each other (two monopoles).

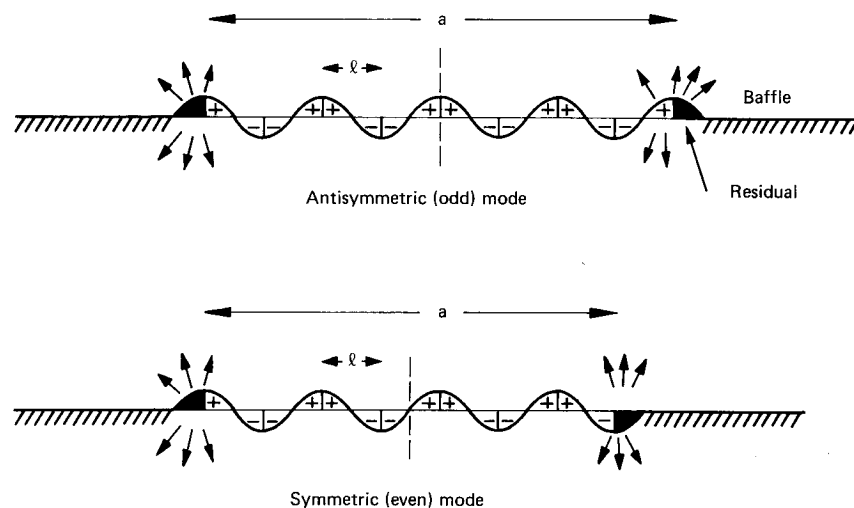


Figure 13. Local cancellation on vibrating plates below coincidence, top) odd mode – uncanceled segments are in phase – two monopoles; bottom) even mode – uncanceled segments are out of phase, forming a dipole which is not as efficient a radiator as two monopoles

## 9.12 Damping of Panels

Elastomeric layers are commonly laminated to vibrating structures to provide additional damping. They dissipate energy when then are flexed (or bent). Automobile undercoating is a good example of materials of this type. Adhesive backed damping sheets are commercially available. The effectiveness of the damping layer is quantified by its loss factor,  $\eta$ ,

The loss depends on the thickness of the damping layer relative to the thickness of the structure as seen in Figure 15. These materials tend to be highly sensitive to temperature.

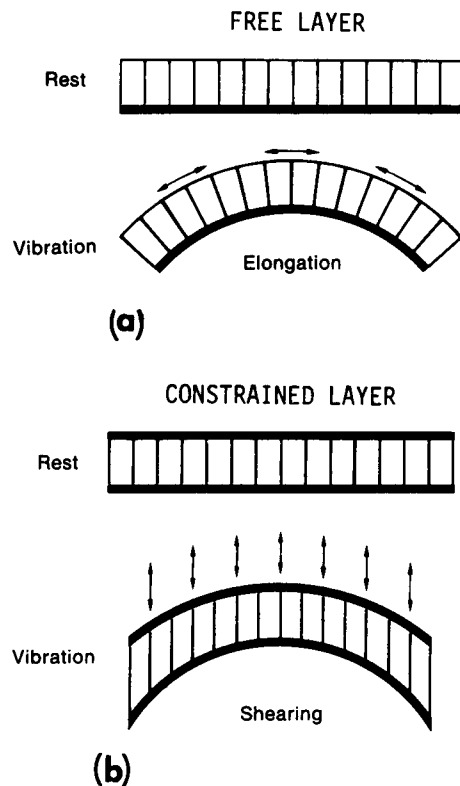


Figure 14. The different stress states in free layer and constrained layer damping: a) tensile strain, predominantly at the outer edge b) shear strain, uniform throughout layer

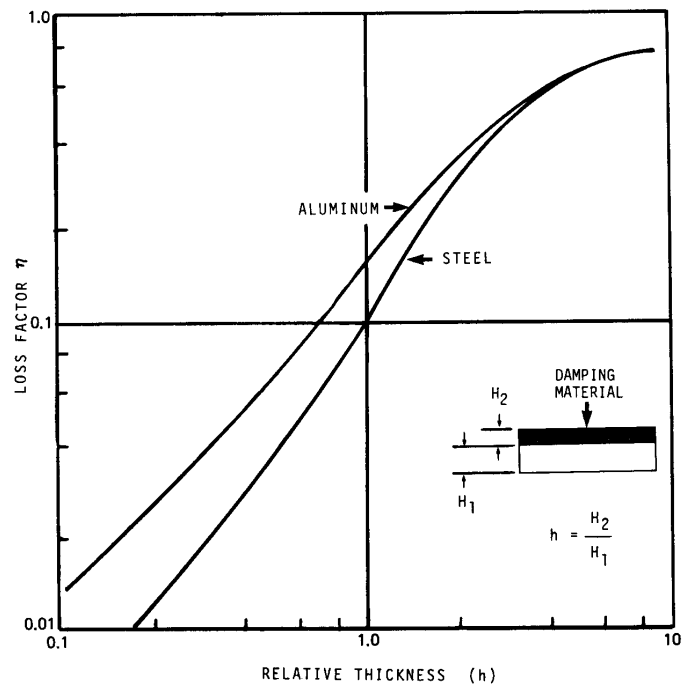


Figure 15. The effect of free layer damping thickness on the loss factor of aluminum and steel plates at 20 degrees C.

### Design Guidelines for Panel Damping:

- A damping layer is effective if the structure is vibrating near resonance **and** the mode is an efficient radiator
- For  $f > f_c/2$ , place damping at regions of maximum amplitude (where structure is undergoing the most bending). Placing damping in the vicinity of a structural node will have little effect since no bending (no flexure) is occurring there.
- For  $f < f_c/2$ , place damping at mounting and structural attachment points

- Cover 40% of structural wavelength – free layer damping
- Cover 60% of structural wavelength – constrained layer damping
- Free layer guidelines:
  - A thin layer,  $\frac{1}{2} t$  or 10% of weight will eliminate the “ring”
  - Use 2 to 3 times the thickness of the structure to achieve loss factor  $\eta$  from .3-.6